AP Calculus AB

Unit 3 – Introduction to Differentiation

AP Calculus AB – Worksheet 20 **Average Rates of Change**

1	Find the average rate of change of $f(x)=x^3+1$ over the interval $[2,3]$.
2	Find the average rate of change of $f(x)=\sqrt{4x+1}$ over the interval $[2,12]$.
3	Find the average rate of change of $f(x)=2+\cos x$ over the interval $[0,\pi]$.
4	The amount of money in a bank account, called the *balance,* after *t* years is $f(t)=100(1.08)^t$ dollars. a) What are the units of the rate of change of $f(t)$. b) Find the average rate of change of the balance over the time interval $[0,0.5]$. Explain the meaning of this value in the context of the bank account.
5	The amount of money in a bank account, called the *balance,* after *t* years is $f(t)=200(0.94)^t$ dollars. Find the average rate of change of the balance over the time interval $[0,0.5]$. Explain the meaning of this value in the context of the bank account.
6	The table below shows the weight, in lbs., of a grown man over a certain time period, *t*, in years. Consider the first year of the data collection period as $t=0$. Year: 0, 2, 5, 6, 10 Weight: 145, 160, 175, 160, 185 a) Find the average rate of change of the man's weight over the time period $[0,10]$, using correct units. b) Estimate the instantaneous rate of change of the man's weight at year four using correct units.
7	Evaluate $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ for a) $f(x)=x^2-4x+5$ b) $f(x)=2x+5$
8	Find the equation of the line that passes through $(2,4)$ and is perpendicular to $2x-4y=7$.
9	Verify that $f(x)=\begin{cases}x^2+x-3, & x\le 2\\ 2x+1, & x>2\end{cases}$ is either continuous or discontinuous at $x=2$.

AP Calculus AB – Worksheet 21 **Limit Definition of the Derivative and the Tangent Line Problem**

Find the derivative of the function using the limit process.

Remember: The derivative is a new function, so it requires new notation; Example: $f'(x)$ or $\frac{dy}{dx}$ or y'

1	$g(x)=-5$
2	$h(x)=\frac{3}{2}x+3$

Find the slope of the tangent line to the graph of the function at the given point.

3	$f(t)=5t-t^2;\quad (1,2)$
4	$f(x)=7-9x^2\ ;\ (1,-2)$

Find the equation of the tangent line to the graph of the function at the given value of *x*.

5	$f(x)=x^2+2x+1,\ x=-4$

6	What is the value of k if $f(x)=\begin{cases}2kx^2-x, & x<3\\ x^3+x, & x\geq 3\end{cases}$ is continuous at all real numbers?

The table below shows selected values for a function, $f(x)$, at various values of *x*.

x	0	1	2	5	9
$f(x)$	14	18	24	32	44

7) Find the average rate of change of the function over the interval $[1,2]$.

8) Find the average rate of change of the function over the interval $[5,9]$.

9) Estimate the slope of the function when $x=3$.

AP Calculus AB - Worksheet 22 **Power Rule**

Nearly all men can stand adversity, but if you want to test a man's character, give him power. - Abraham Lincoln

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Review

Find the value of *a* so that the function $f(x)=\begin{cases}4-x^2, & x<-1\\ ax^2-3, & x\geq -1\end{cases}$ is continous. **Show all necessery work.**

Use the Power Rule to find the **derivative** of each function.

1) $y=x^5$	2) $f(x)=4x^3$	3) $f(x)=3x^2-4x+1$
4) $y=5x-1$	5) $f(t)=-2t^2+3t-6$	6) $f(x)=\frac{1}{x^7}$
7) $f(x)=2x^{-1}+5x^2$	8) $f(x)=\sqrt[4]{x}$	9) $y=\sqrt[3]{x^2}$

Find the **instantaneous rate of change** of each function when $x=1$.

10) $y=\frac{1}{3}x^{-3}$	11) $f(x)=8$	12) $y=4x^{-2}-8x+1$
13) $f(x)=\frac{x^{-4}}{4}-\frac{x^{-3}}{3}+\frac{x^{-2}}{2}-x^{-1}+3$	14) $y=2\sqrt{x}-\frac{1}{\sqrt{x}}$	15) $f(x)=x^{4/5}+x^{2/3}-3$

Find the **slope of the curves** at the indicated values of *x*.

16) $y=x^4-3x^2+2, \quad x=2$	17) $f(x)=x^3+x, \quad x=-1$
18) $f(x)=\frac{2}{\sqrt[4]{x^3}}, \quad x=1$	19) $y=x^3+3x^2+2x, \quad x=0$

20) Evaluate $\lim\limits_{\Delta x\to 0}\frac{(x+\Delta x)^2-5(x+\Delta x)+6-(x^2-5x+6)}{\Delta x}$ using the **power rule shortcut**. (Make connections)

21) Find two unique functions, $f(x)$, such that both of their derivatives are $f'(x)=x^2+2x+8$. (Answers will vary)

AP Calculus AB - Worksheet 23 **Tangent and Normal Lines (Power Rule)**

For each of the following:

a) Graph the curve.
b) Find the equation of the tangent line at the given point. Graph the tangent line.
c) Find the equation of the normal to the curve at the given point. Graph the normal line.

1. $y = x^2 - 3,\ (2,1)$ 2. $f(x) = \sqrt{x},\ (4,2)$ 3. $y = 2 - 4x^{-2},\ (2,1)$

4. Find the equation of the tangent line to the curve $f(x) = 4x^2 + 8x + 1$ at $x = 1$.

5. Find the equation of the normal line to the curve $f(x) = 3x^2 - x^3$ at $x = -2$.

6. Find the point(s) on the curve $f(x) = 2x^3 + 3x^2 - 12x + 1$ where the tangent is horizontal.

7. Find all values of x for which the graph of $g(x) = x^3 - 17x^2 + 63x$ have horizontal tangent lines.

8. For what point on the curve of $f(x) = 8x^2 - 7x$ is the slope of the tangent line equal to -87.

9. Find an equation of the tangent line to the curve $y = x^{\frac{3}{2}}$ that is parallel to the line $y = 1 + 3x$.

10. **Challenge Problem** Find a parabola with equation $f(x) = ax^2 + bx + c$ such that:

- $f(2) = 15$
- $f'(1) = 4$
- $f'(-1) = -8$

(Hint: You will create 3 different equations in order to solve for *a*, *b*, and *c*.)

11. Evaluate $\lim_{h \to 0} \dfrac{(x+h)^3 - 2(x+h) - (x^3 - 2x)}{h}$ using the Power Rule.

AP Calculus AB - Worksheet 24 **The Package Rule**

For problems 1-14, find $f'(x)$. Do not simplify your answer.

1. $f(x)=(3x+2)^8$

2. $f(x)=(1-x)^6$

3. $f(x)=\left(1-2x^2\right)^3$

4. $f(x)=\left(x^2+3x+1\right)^5$

5. $f(x)=\dfrac{1}{\left(x^2+2\right)^3}$

6. $f(x)=\sqrt{x+1}$

7. $f(x)=\left(2x^2-3x+1\right)^4$

8. $f(x)=\sqrt{x^2+2x-1}$

9. $f(x)=\dfrac{1}{\sqrt[3]{x^3+3}}$

10. $f(x)=\left(x^2-4\right)^{-\frac{1}{2}}$

11. $f(x)=6x^4-8x^3$

12. $f(x)=\dfrac{2}{3x-5}$

13. $f(x)=\left(-9x+\sqrt{x}\right)^{-2}$

14. $f(x)=\dfrac{1}{x^5}$

15. $f(x)=\left(4x^6-5x^5+5\right)^{23}$

Write the equation of the normal line at the given *x*-value.

16. $f(x)=2x^3-2x-7$ when $x=-1$

17. $f(x)=(2x+3)^3$ when $x=0$

18. $f(x)=\sqrt{x-1}$ when $x=5$

19. $f(x)=\dfrac{2}{x^3}$ when $x=2$

AP Calculus AB - Worksheet 25 **Product and Quotient Rules for Derivatives**

Differentiate the following functions. **Do not simplify the answer**

1. $g(t) = 6t^{\frac{5}{3}}$

2. $B(x) = \dfrac{8x^2 - 6x + 11}{x - 1}$

3. $G(v) = \dfrac{v^3 - 1}{v^3 + 1}$

4. $f(s) = 15 - s - 4s^2 - 5s^4$

5. $f(x) = \dfrac{1}{1 + x + x^2 + x^3}$

6. $M(x) = \dfrac{2x^3 - 7x^2 + 4x + 3}{x^2}$

7. $f(x) = 3x^2 + \sqrt[3]{x^4}$

8. $p(x) = 1 + \dfrac{1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x^3}$

9. $g(x) = x^4 - \sqrt[4]{x^3}$

10. $h(x) = (5x - 4)^2$

11. $k(x) = (2x^2 - 4x + 1)(6x - 5)$

12. $F(t) = t^2 + \dfrac{1}{t^2}$

13. $f(x) = \dfrac{4x - 5}{3x + 2}$

14. $N(z) = \dfrac{4z^2}{3z + 2}$

15. $k(r) = r^3(3r^4 - 7r^2 + 2r)$

16. $S(x) = \dfrac{1}{(6x + 5)^3}$

17. The function *f* is defined by $f(x) = \dfrac{x}{x + 2}$. Determine the points (x, y) on the graph of *f* for which the slope of the tangent line to *f* at (x, y) has slope $\dfrac{1}{2}$.

AP Calculus AB – Worksheet 26 **Estimating Derivatives Using the Calculator**

Estimate the derivative at the given point by using a calculator.

1	$f(x)=x\sqrt{2-x};$ find $f'(-10)$
2	$f(x)=\cos(5x);$ find $f'(2)$
3	$f(x)=\ln\left(\sqrt{x}\right);$ find $f'(1)$
4	$f(x)=e^{\frac{x}{3}};$ find $f'(4)$
5	$f(x)=\tan(\sin x);$ find $f'(-3)$
6	$f(x)=2^{\ln x};$ find $f'(2)$
7	The function $f(t)=\dfrac{t}{\cos t}$ measures the height of a bird in meters where *t* is seconds. Find $f'(2)$.
8	The function $f(t)=\sin^2 t$ measures the depth of a submarine measured in feet where *t* is minutes. Find $f'(12.5)$
9	For $f(x)=\cos(\tan x)$, find the equation of the tangent line and normal line at $x=2$.
10	For $f(x)=\dfrac{x^4}{\sqrt{x}}$, find the equation of the tangent line and normal line at $x=3$.

AP Calculus AB – Worksheet 27 **Derivatives Using Tables**

For each problem, you are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

1

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	−1	1	2
2	4	−1	3	$\frac{3}{2}$
3	3	−1	4	1
4	2	−1	5	1
5	1	0	6	$-\frac{1}{2}$
6	2	1	4	−2

a) Given $h(x)=f(x)+g(x)$, find $h'(2)$

b) Given $s(x)=f(x)-g(x)$, find $s'(3)$

c) Given $p(x)=f(x)\cdot g(x)$, find $p'(4)$

d) Given $q(x)=\dfrac{g(x)}{f(x)}$, find $q'(5)$

e) Given $c(x)=(f(x))^2$ find $c'(2)$

f) Given $m(x)=f(g(x))$, find $m'(6)$

2

Let h(x) = f(g(x)) and p(x) = g(f(x)). Use the table below to compute the following derivatives.

a. $h'(4)$
b. $p'(3)$

x	**1**	**2**	**3**	**4**
f(x)	3	2	4	1
f′(x)	− 9	− 3	− 2	− 8
g(x)	4	1	3	2
g′(x)	$\frac{1}{5}$	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{2}{5}$

3

$f(3)$	$g(3)$	$f'(3)$	$g'(3)$
−1	2	5	−2

The table above gives values for the functions f and g and their derivatives at $x = 3$. Let k be the function given by $k(x) = \dfrac{f(x)}{g(x)}$, where $g(x) \neq 0$. What is the value of $k'(3)$?

4

For a given function $f(x)$, it is known that $f(4)=5$ and $f'(4)=4$. Let $g(x)=x^2\cdot f(x)$ and $h(x)=\dfrac{f(x)}{x-5}$.

a. Find an equation of the line that is tangent to the graph of $y=g(x)$ at $x=4$.

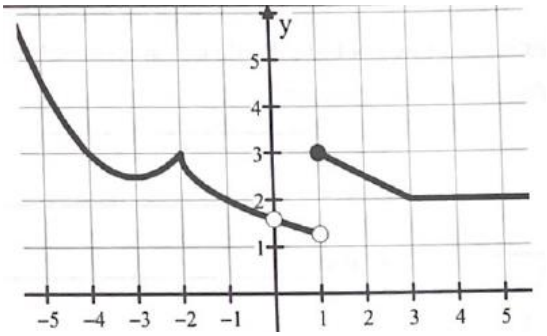
b. Find an equation of the line that is tangent to the graph of $y=h(x)$ at $x=4$.

AP Calculus AB – Worksheet 28 **Differentiability**

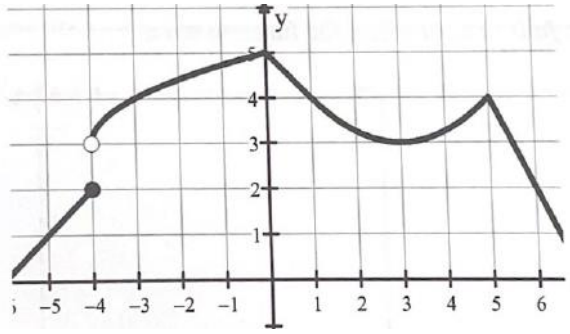
Identify any values of *x* for which the graph of $f(x)$ is:

a) Not continuous
b) Continuous but not differentiable

1)



2)



Determine if the function is differentiable at the given value.

3)

$$f(x)=\begin{cases} x, & x\le 1 \\ x^2, & x>1 \end{cases}, \quad x=1$$

4)

$$f(x)=\begin{cases} x^2-1, & x\le 2 \\ 4x-5, & x>2 \end{cases}, \quad x=2$$

5) Sketch the graph of a continuous function such that:

$f(0)=2;$

$f'(x)=-3$ for $-\infty<x<\infty$

6) Sketch the graph of a continuous function such that:

$f(0)=4$

$f'(0)$ is undefined

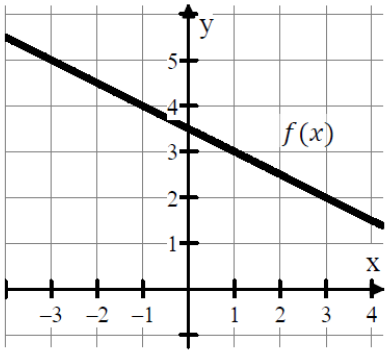
$f'(x)<0$ for $x<0$

$f'(x)>0$ for $x>0$

7)

The graph of the function f is shown at the right.

The function h is defined by $h(x)=f(2x^2-x)$. Find the slope of the line tangent to the graph of h at the point where $x=-1$



AP Calculus AB – Worksheet 29 **Derivatives (Power, Package, Product and Quotient Rules) Review**

1	Use the limit definition of the derivative to find $f'(x)$ for $f(x)=2x^2+1$

Find the derivative of each function below. Simplify your answer.

2	$y=\dfrac{2-x}{3x+1}$
3	$y=\dfrac{2}{(5x+1)^3}$
*4	$f(x)=\dfrac{x}{\sqrt{1-x^2}}$
5	$f(x)=(x^2-2)(x^{-1}+2)$
6	$y=3x^2+\dfrac{2}{x}-\dfrac{5}{x^2}$

Answer each question about tangents and normals.

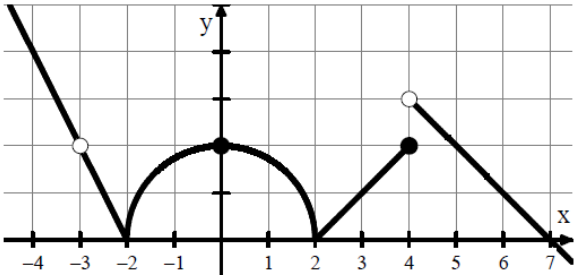
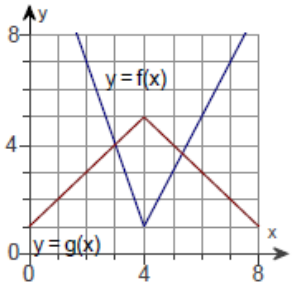
7	Find the points on the curve $y=2x^3-3x^2-12x+20$ where the tangent line is parallel to the x-axis.
8	Find the slope of the normal to $f(x)=2x^3+x^2-1$ at the point where $x=\dfrac{1}{2}$
9	Find the equation of the tangent to $y=\sqrt{x^3+1}$ at the point where $x=2$.
10	Find $\dfrac{dy}{dx}$ for $y=(x^2+1)(x^3+1)$, then find the slope of the normal when $x=-1$.
11	If the line $y=2x-3$ is tangent to the function $f(x)=x^2+2x-3$, what is the point of tangency?
12	At what x-value is $y=3x-1$ tangent to $f(x)=x^3+1$.

For #13-18, use the table below to find the indicated value.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	1	2	2
2	2	1	4	$\dfrac{3}{2}$
3	3	1	5	$-\dfrac{1}{2}$
4	4	1	3	$-\dfrac{3}{2}$
5	5	1	2	-1

13. Given $h(x)=f(x)+g(x)$, find $h'(2)$.
14. Given $d(x)=f(x)-g(x)$, find $d'(3)$.
15. Given $p(x)=f(x)\cdot g(x)$, find $p'(3)$.
16. Given $q(x)=\dfrac{g(x)}{f(x)}$, find $q'(3)$.
17. Given $k(x)=(f(x))^2$, find $k'(1)$.
18. Given $c(x)=f(g(x))$, find $c'(1)$

AP Calculus AB – Worksheet 30 **Unit 3 Review**

1	Find the equations of the tangent and normal lines to the curve $f(x)=7\sqrt{x}$ at $x=4$.
2	Find $\frac{dy}{dx}$ for $y=\frac{7x-1}{4x+5}$. **Simplify**
3	For what value(s) of *x* does $f(x)=x^3-7x^2+8x+2$ have horizontal tangents?
4	Find the derivative of $f(x)=(6x^3-x+1)(9-x^6)$.
5	Find the slope of the tangent line to the curve $f(x)=\frac{-3x^2+2x+3}{(x-2)^2}$ at the point where $x=-2$.
6	For what point on the curve of $y=8x^2+4x$ is the slope of the tangent line equal to -44?
7	Find the derivative of $y=(8x^3+7)^{3/2}$
8	Differentiate $f(x)=\frac{1}{(6x+7)^5}$.
9	The figure below shows the graph a function $f(x)$.  a) At what values of *x* does $f(x)$ appear to be continuous? b) At what values of *x* does $f(x)$ appear to be differentiable?
10	Assume that $f'(2)=-3$, $g'(4)=\frac{2}{5}$, $g(4)=2$, and $y=f(g(x))$, what is the value of $\frac{dy}{dx}$ at $x=4$?
11	 Let $H(x)=f(x)+g(x)$, where the graphs of *f* and *g* are shown in the figure to the left. Find $H'(3)$.